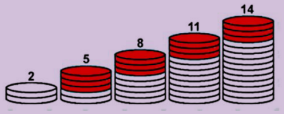
Book 3

Julia Robinson Mathematics Festival®

Julia Robinson
(1919 - 1985)

Good Problems



Arithmetic Series

Tax Man

Sum of 51

105

How Many 7's

Egg Timer

Skyscrapers

Marching Band

"The JRMF really gets it right. Usually the best parts of mathematics are kept away from the public, as if you needed to be a mathematician to get to the fun stuff! It's refreshing to see a festival that brings this stuff to light, and in such a relaxed atmosphere. If you're lucky enough to have a JRMF near you, don't miss it! It's the best math party around."

– Vi Hart, Mathemusician, youtube.com/user/ViHart

Festival activities are designed to open doors to higher mathematics for students in grades K–12. Visit **www.JRMF.org** for more information about Julia Robinson Mathematics Festivals.

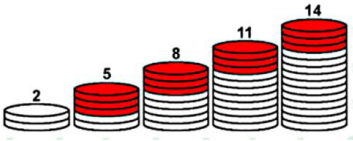
Compiled by Peter Liljedahl.

The problems in this booklet were selected from a variety of resources by Peter Liljedahl, who is Professor in the Faculty Education at the Simon Fraser University in British Columbia, Canada. These and many other problems can be found on Peter's website: **www.peterliljedahl.com/teachers/good-problem**. Solutions aren't included in this booklet, but if you follow the link to the source, you may find some solutions as well as additional similar problems.

At Julia Robinson Mathematics Festivals, you will find many more interesting problems that you can explore with friends. To find a festival, visit our website: **http://jrmf.org/register.php**. If you don't find a festival convenient to you, consider hosting your own festival. Email **info@jrmf.org** if you are interested in volunteering, organizing, or hosting a festival.

1. Arithmetic Series

Find the sum of the arithmetic series 13 + … + 61.
An arithmetic series is a sequence of numbers such that the difference between the consecutive terms is constant.
For instance, the sequence 2, 5, 8, 11, 14, ... is an arithmetic progression with common difference of 3.

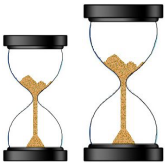


2. 105

In how many ways can 105 be expressed as the sum of at least two consecutive positive integers? [from the Canadian Team Math Contest, 2014, #12] For more problems like this, check out the Trapezoidal Number activity in the JRMF *Sample of Mathematical Puzzles BOOK 1* (page 6) and the Staircase Numbers activity in the *Recreational Mathematics (really!) BOOK 2* (page 5). Email **info@jrmf.org** if you want us to send you the Trapezoidal Number or Staircase Number activities.

3. Egg Timer

How do you make a 9-minute egg if all you have is a 4-minute and 7-minute egg timer? If you come up with one solution, see if you can come up with other solutions.



4. Tax Man

Tax Man is played like this: Start with a collection of paychecks, from $1 to $12. You can choose any paycheck to keep. Once you choose, the tax collector gets all paychecks remaining that are factors of the number you chose. The tax collector must receive payment after every move. If you have no moves that give the tax collector a paycheck, then the game is over and the tax collector gets all the remaining paychecks. The goal is to beat the tax collector.

Example:

TURN 1: Take $8. The tax collector gets $1, $2 and $4.

TURN 2: Take $12. The tax collector gets $3 and $6 (the other factors have already been taken).

TURN 3: Take $10. The tax collector gets $5.
You have no more legal moves, so the game is over, and the tax collector gets $7, $9 and $11, the remaining paychecks.

Total Scores:

You: $8 + $12 + $10 = $30.

Tax Collector: $1 + $2 + $3 + $4 + $5 + $6 + $7 + $9 + $11 = $48.

Questions:

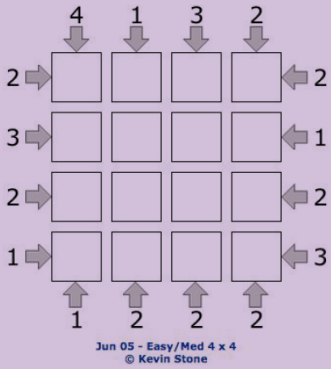
- Is it possible to beat the tax collector in this $12 game? If so, how?
- What is the maximum score you can get?
- BONUS: What if you played the game with paychecks from $1 to $24? How about $1 to $48?

For more bonus questions, check out the JRMF Divisor Game activity available online at **http://jrmf.org/problems/DivisorGame.pdf**. It offers several fun explorations of how divisors of numbers are related to each other.

[Tax Man problem from Dan Finkel, New York Times: **http://wordplay.blogs.nytimes.com/2015/04/13/finkel-4/**]

Cover image: Bingo Number 51
https://www.flickr.com/photos/lwr/2455243440

5. Skyscrapers



- Complete the grid such that every row and column contains the numbers 1 to 4.
- Each row and column contains each number only once.
- However, you have to follow the rules around the outside, which tell you how many skyscrapers you can see.
- You can't see a shorter skyscraper behind a taller one.

Visit Skyscrapers online (**www.brainbashers.com/skyscrapers.asp**) for more of the same as well as the option to work on the problems online.

6. Sum of 51

How many 6 digit numbers are there whose digits sum to 51? This problem is #7 in Practice Fermat Number 4. For more problems involving sums of digits, check out the Digit Sums and Graphs activity available in the JRMF *Sample of Mathematical Puzzles BOOK 1* and the Subdivisions of Numbers activity, both of which may be requested at your local JRMF.

7. How Many 7's

If you write out the numbers from 1 to 1000, how many times will you write the number 7?

8. Marching Band

Students in a marching band want to line up for their performance. The problem is that when they line up in 2's there is 1 left over.
When they line up in 3's there are 2 left over.
When they line up in 4's there are 3 left over.
When they line up in 5's there are 4 left over.
When they line up in 6's there are 5 left over.
When they line up in 7's there are no students left over. How many students are there?

This problem, suggested by John Grant McLoughlin. This problem and many other wonderful problems can be found in *Teaching Through Problems Worth Solving - Grade 8*, which is available online.